Introduction to basic Ideas.

Combinatorial maths concern with counting the number of ways of arranging the given objects in a prescribed way.

Problem

Suppose that each of k distinguish balls has to be coloured with anyone of n given colours. The Problem is to find given howmany different colouring are possible.

Let x denote the number of balls coloured with the first colour and

x_2 denote the number of balls coloured with the Second colour.

The required number is the number of solutions of the equations.

$$x_1 + x_2 + \dots + x_n = k.$$

where $x_1, x_2 \dots x_n$ is the non-negative integers

As this numbers will depend upon both n and x it is denoted by F(n, k)

Special cases

Case : 1

There is only one colour available (n-1). The k balls can be colouring only one way.

Thus $F(1, k) = 1 \; \forall k \geq 1$

case : 2

If there are n colours but only one ball there are n possible colourings

Thus $F(n, 1) = n. \; \forall n \geq 1$

case : 3

Suppose $n = k = 2$. If the colours are black write the balls can be coloured both black both white or one of each

Thus $F(2, 2) = 3$

To find general formula for F(n, k)

Approach : 1

consider F(n, k) and concentrate or the nth colour when the k balls are coloured this nth colour may or may not be used. If it is not uses there are only $(n-1)$ colours and the colours can be performed $F(n-1, k)$ ways.

If the n^{th} colour is used one ball can be coloured by a and then remove to $k-1$ balls. which can be coloured in $F(n, k-1)$ ways.

Thus $F(n,k) = F(n-1, k)$,
$F(n, k-1)$

Problem : 1

Find $F(3,2)$ using first approach?

Soln:-

$$F(3,2) = F(2,2) + F(3,1)$$
$$= 3+3 = 6.$$

Problem: 2

Find $F(4,2)$

Soln:-

$$F(4,2) = F(3,2) + F(4,1)$$
$$= F(2,2) + F(3,1) + F(4,1)$$
$$= 3+3+4$$
$$= 10.$$

Problem: 3

Find $F(5,3)$

Soln:-

$$F(n,k) = F(n-1,k) + F(n,k-1)$$

$$\begin{aligned}
F(5,3) &= F(4,3) + F(5,2) \\
&= F(3,3) + F(4,2) + F(4,2) + F(5,1) \\
&= F(2,3) + F(3,2) + F(3,2) + F(4,1) + \\
&\quad F(3,2) + F(4,1) + F(5,1) \\
&= F(1,3) + F(2,2) + F(2,2) + F(3,1) + \\
&\quad F(2,2) + F(3,1) + F(4,1) + F(2,2) + \\
&\quad F(3,1) + F(4,1) + F(5,1) \\
&= 1 + 3 + 3 + 3 + 3 + 3 + 4 + 3 + 3 + 4 + \\
&\quad 5 \\
&= 35.
\end{aligned}$$

Problem: 4

Find $(4,3)$ using first approach

Soln:-

$$F(n,k) = F(n-1,k) + F(n,k-1)$$

$$\begin{aligned}
F(4,3) &= F(3,3) + F(4,2) \\
&= F(2,3) + F(3,2) + F(3,2) + \\
&\quad F(4,1) \\
&= F(1,3) + F(2,2) + F(2,2) +
\end{aligned}$$

$F(3,1)+F(2,2)+F(3,1)+F(4,1)$

$= 1+3+3+3+3+3+4$

$= 20.$

Problem : 5

Find $F(10,3)$ using first approch

Soln:-

$F(10,3) = F(9,3) + F(10,2)$

$= F(8,3) + F(9,2) + F(9,2) + F(10,1)$

$= F(7,3) + F(8,2) + F(8,2) + F(9,1) + F(8,2) + F(9,1) + F(10,1)$

$= F(6,3) + F(7,2) + F(7,2) + F(8,1) + F(7,2) + F(8,1) + F(9,1) + F(7,2) + F(8,1) + F(9,1) + F(10,1)$

$= F(5,3) + F(6,2) + F(6,2) + F(7,1) + F(6,2) + F(7,1) + F(8,1) + F(6,2) + F(7,1) + F(8,1) + F(9,1) + F(6,2) + F(7,1) + F(8,1) + F(9,1) + F(10,1)$

$= F(4,3) + F(5,2) + F(5,2) + F(6,1) + F(5,2) + F(6,1) + F(7,1) + F(5,2) + F(6,1) + F(7,1) + F(8,1) + F(5,2) + F(6,1) + F(7,1) + F(8,1) + F(9,1) + F(5,2) + F(6,1) + F(7,1) + F(8,1) + F(9,1) + F(10,1)$

$= F(3,3) + F(1,2) + F(1,2) + F(1,2) + F(1,2) +$
$F(1,2) + F(1,2) + F(1,2)$
$= 5+5+6+5+6+7+5+6+7+8+5+6+7+8+9+5+$
$6+7+8+9+10$

$F(3,3) = 10$

$F(1,2) = 10$

$= 10+10+10+10+10+10+10+10+40$

$= 220$

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Problem 6
Find $F(9,4)$ using the first approach

$$f(9,4) = f(8,4) + f(9,3)$$
$$= f(7,4) + f(8,3) + f(8,3) + f(9,2)$$
$$= f(6,4) + f(7,3) + f(7,3) + f(8,2) + f(7,3) + f(8,2) + f(8,2) + f(9,2)$$
$$= f(5,4) + f(6,3) + f(6,3) + f(7,2) + f(6,3) + f(7,2) + f(7,2) + f(8,1) + f(6,3) + f(7,2) + f(7,2) + f(8,1) + f(7,2) + f(8,1) + f(8,1) + f(9,1)$$
$$= f(4,4) + f(5,3) + f(5,3) + f(6,2) + f(5,3) + f(6,2) + f(6,2) + f(7,1) + f(5,3) + f(6,2) + f(6,2) + f(7,1) + f(6,2) + f(7,1) + f(7,1) + f(8,1) + f(5,3) + f(6,2) + f(6,2) + f(7,1) + f(6,2) + f(7,1) + f(7,1) + f(8,1) + f(6,2) + f(7,1) + f(7,1) + f(8,1) + f(7,1) + f(8,1) + f(8,1) + f(9,1)$$
$$= f(3,4) + f(4,3) + f(4,3) + f(5,2) + f(4,3) + f(5,2) + f(5,2) + f(6,1) + f(5,2) + f(6,1) + f(4,3) + f(5,2) + f(5,2) + f(6,1) + f(5,2) + f(6,1) + f(7,1) + f(4,3) + f(5,2) + f(5,2) + f(6,1) + f(7,1) + f(5,2) + f(6,1) + f(5,2) + f(6,1) + f(5,2) + f(5,2) + f(6,1) + f(8,1) + f(4,3) + f(7,1) + f(5,2) + f(5,2) + f(6,1) + f(8,1) + f(5,2) + f(6,1) + f(7,1) + f(8,1) + f(9,1) + f(6,1) + f(7,1) \quad (1)$$

$f(3,4) = f(2,4) + f(3,3)$

$= f(1,4) + f(2,3) + f(2,3) + f(3,2)$

$= f(1,4) + f(1,3) + f(2,2) + f(1,3) + f(2,2)$

$+ f(2,2) + f(3,1)$

$= f(1,4) + f(1,3) + f(1,2) + f(2,1) + f(1,3) +$

$f(1,2) + f(2,1) + f(3,1)$

$= 1 + 1 + 1 + 2 + 1 + 1 + 2 + 1 + 2 + 3$

$= 15$

$f(4,3) = f(3,3) + f(4,2)$

$= f(2,3) + f(3,2) + f(3,2) + f(4,1)$

$= f(1,3) + f(2,2) + f(2,2) + f(3,1) + f(4,1)$

$+ f(2,2) + f(3,1)$

$= 1 + 3 + 3 + 3 + 4 + 3 + 3$

$= 20$

$f(5,2) = f(4,2) + f(5,1)$

$= f(3,2) + f(4,1) + f(5,1)$

$= f(3,2) + f(3,1) + f(4,1) + f(5,1)$

$= f(2,2) + f(3,1)$

$= 3 + 3 + 4 + 5$

$= 15$

$= 15 + 20 + 14 + 11 + 20 + 11 + 11 + 6 + 20 + 15 + 15 + 6$

$15 + 15 + 6 + 15 + 6 + 7 + 15 + 6 +$

$+ 15 + 6 + 7 + 20 + 15 + 15 + 6 + 15 + 6 + 7 + 15 +$

$7 + 8 + 20 + 15 + 15 + 6 + 15 + 6 + 7 + 8 + 9$

$6 + 7 + 8 + 15 + 6 + 7 + 8 + 9$

$f(9,4) = 495$

Problem 1

Solve the recurrence relation $a_n = n a_{n-1}$ with the boundary condition $a_1 = 1$

Soln:

$a_n = n a_{n-1}$ and

similarly $a_{n-1} = (n-1) a_{n-2}$

$a_{n-2} = (n-2) a_{n-3}$

so that

$$a_n = n a_{n-1}$$
$$= n(n-1) a_{n-2}$$
$$= n(n-1)(n-2) a_{n-3}$$
$$= n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$
$$\therefore a_n = n!$$

for example $a_3 = 3! = 6$

$a_4 = 4! = 24$

Approach 2

Consider the expression

$(1+x+x^2+\cdots)(1+x+x^2+\cdots)\cdots(1+x+x^2+\cdots)$ where there are n brackets

Each powers of x are obtained Each powers occuring ~~are~~ a number of times.

The above expression that x^k is obtained by choosing the term x^t in the first bracket x^t in the second bracket and so on with the condition that

$$t_1 + t_2 + \cdots + t_n = k$$

$f(n,k)$ coefficient of x^k in

$(1+x+x^2+\ldots)^n$ But $(1+x+x^2+\ldots) = (1-x)^n$

$f(n,k)$ = coefficient of x^n in

$$1 + \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} + \frac{n(n+1)(n+2)x^3}{3!} + \ldots + n(n+1)(n+2)\ldots\frac{(n+k-1)}{k!}x^k$$

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problem 2

solve the recurrence relation $a_n = n^2 a_{n-1}$ with boundary condition $a_1 = 1$

soln:

$$a_{n-1} = (n-1)^2 a_{n-2}$$

$$a_{n-2} = (n-2)^2 a_{n-3}$$

$$\begin{aligned} a_n &= n^2 a_{n-1} \\ &= n^2 (n-1)^2 a_{n-2} \\ &= n^2 (n-1)^2 (n-2)^2 a_{n-3} \\ &= n^2 (n-1)^2 (n-2)^2 \ldots 2^2 a_1 \\ &= n^2 (n-1)^2 (n-2)^2 \ldots 2^2 \ldots 1 \\ &= (n(n-1)(n-2)\ldots 2.1)^2 \\ &= (n!)^2 \end{aligned}$$

Problem 3

solve the recurrence relation $a_n = a_{n-1} + n$ with boundary condition

i) $a_1 = 1$ ii) $a_1 = 0$

Soln:

i) $a_n = a_{n-1} + n$

$a_{n-1} = a_{n-2} + (n-1)$

$a_{n-2} = a_{n-3} + (n-2)$

$\vdots$

$a_n = a_{n-1} + n$

$= a_{n-2} + (n-1) + n$

$= a_1 + 2 + \cdots + (n-2) + (n-1) + n$

case (i)

If $a_1 = 1$

$a_n = 1 + 2 + \cdots + (n-2) + (n-1) + n$

$\therefore a_n = \frac{n(n+1)}{2}$

case (ii)

If $a_1 = 0$

$a_n = 2 + \cdots + (n-2) + (n-1) + n$

$\therefore a_n = \frac{n(n+1)}{2} - 1$

1. Suppose a row of n cages in given and it is required to place k Indistinguishablo tions in them so that no cage contains more than one lion and no two consecutive cages both contained a lion Let $g(n,k)$ denote the no.of ways in which this can be done Prove that

i) $g(2k-1,k)=1$

ii) $g(n,k)=0$ if $n<2k-1$

iii) $g(n,1)=n$

iv) $g(n,k)=g(n-2,k-1)+g(n-1,k)$ if $k\geq 2$

Deduce that

v) $g(6,3)=4$

vi) $g(2k,k)=g(2k-2,k-1)+1$

vii) $g(2k,k)=k+1$

Soln

i) To prove $g(2k-1,k)=1$

to prove the no. of place ink lions in $(2k-1)$ cages

we have to place the first lion the 1^{st} cage and 2^{nd} lion in the 3^{rd} cages 3^{rd} lion in the 5^{th} cage and so on.

This can be done in only one way $\therefore g(2k-1,k)=1$

ii) To prove

$g(n,k)=0$ if $n<2k-1$

There are n cages and k lions.

To prove k lions we need at least $(2k-1)$ cages but given $n<2k-1$.

This is not passible

$$g(n,k)=0$$

iii) $g(n,1)=n$

There are one lion and n cages we can place this one lion in n cages in n ways

$$g(n,1)=n$$

iv) case (i)

suppose the last cages contain a lion. But given no two consencutive cages contain a lion. There are $(n-2)$ cages to place $(k-1)$ lions. This can be done in $g(n-2,k-1)$ ways.

case (ii)

suppose the last cages does not contain a lion.

There are $(n-1)$ cages to place k lions To place this k lions in $(n-1)$ cages This can be done in $g(n-1,k)$ ~~cage~~ ways.

$$\therefore g(n,k)=g(n-2,k-1)+g(n-1,k)$$

v) $g(6,3) = g(4,2)+g(5,3)$

$= g(2,1)+g(3,2)+g(3,2)+g(4,3)$

$= g(2,1)+g(3,2)+g(3,2)+g(4,3)$

$= 2+g(1,1)+g(2,2)+g(1,1)+g(2,2)$

$+ g(2,2)+g(3,3)$

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$= 2+1+0+1+0+0$

$= 4$

vi) $g(2k,k) = g(2k-2, k-1)+1$

$g(2k,k) = g(2k-2, k-1) + g(2k-1, k)$

$g(2k,k) = g(2k-2, k-1) + 1$

vii) $g(2k,k) = k+1$

$g(2k,k) = g(2k-2, k-1)+1$

$= g(6,3)+1$ put $k=4$

$= 4+1 = 5$

$g(2k,k) = k+1$

Find $f(9,4)$ using 2 approach

soln

$$f(9,4) = \frac{(9+3)!}{8!\,4!} = \frac{12!}{8!\,4!}$$

$$= \frac{479001600}{967680}$$

$= 495$

Find $f(10,4)$

soln $f(n,k) = \dfrac{(n+(k-1))!}{(n-1)!\,k!}$

$$f(10,4) = \frac{(10+3)!}{9!\,4!}$$

$= 715$

4 Find $f(10,5)$

Soln

$$f(n,k) = \frac{(n+k-1)!}{(n-1)!\,k!}$$

$$f(10,5) = \frac{14!}{9!\,5!} = 2002$$

Approach - 3

The problem is to colour k balls using n colours.

Let x_1 denotes the no. of balls colour in the first colour and x_2 denote the no of balls coloured in the 2^{nd} colour and so on

This can be represented geometrical as follows

for example

$n = 4, \quad k = 5$

Put two cross at a consecutive marks on a straight line. This cross represented the two balls coloured with the I^{st} colour at the next mark line space o to significant the change of colour.

Then put one cross to represent the one ball of the second colour. Then another 'o' and so on.

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xxoxoxoxox which has five crosses five balls) 3 o's (change of colour)

In the general case with n colour k balls f(n, k) will be the no of such sequence o's and crosses containing k crosses and (n-1) o's Each such sequence contains (n+k-1) symbols

$$f(n,k) = \binom{n+k-1}{k} \text{ or } \binom{n+k-1}{n-1}$$

$$= \frac{(n+k-1)!}{k!(n+k-1-k)!}$$

$$f(n,k) = \frac{(n+k-1)!}{k!(n-1)!}$$

Problem : 2

How many solutions are there in the equation $x+y+z=10$ where x, y, z are non-negative integers.

Soln :

Given $x+y+z=10$

Here, $n=3$, $k=10$

$$f(n,k) = \frac{(n+k-1)!}{(n-1)!\,k!}$$

$$= \frac{(3+9)!}{2!\,10!} = \frac{12!}{2!\,10!}$$

$$= 66.$$

Problem : 3

Prove that $\binom{n}{n-2} = \binom{n}{2} = \frac{1}{2} n(n-1)$.

Soln :

$$\binom{n}{n-2} = \frac{n!}{(n-2)!\,(n-n+2)!}$$

$$= \frac{n!}{(n-2)!\,2!}$$

$$= \frac{n(n-1)(n-2)!}{(n-2)!\,2!}$$

$$= \frac{1}{2} n(n-1).$$

$$\binom{n}{2} = \frac{n!}{(n-2)!\,2!}$$

$$= \frac{n(n-1)(n-2)!}{(n-2)!\,2!}$$

$$= \tfrac{1}{2}\, n(n-1).$$

$$\therefore \binom{n}{n-2} = \binom{n}{2} = \tfrac{1}{2}\, n(n-1).$$

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1. $f(9,4)$.

Soln:

$$f(9,4) = f(8,4) + f(9,3)$$

$$= f(7,4) + f(8,3) + f(8,3) + f(9,2)$$

$$= f(7,4) + 2f(8,3) + f(9,2).$$

$$= f(6,4) + f(7,3) + 2\left[f(7,3) + f(8,2) + f(8,2) + f(9,1)\right]$$

$$= f(6,4) + 3f(7,3) + 3f(8,2) + f(9,1).$$

$$= f(5,4) + f(6,3) + 3\left[f(6,3) + f(7,2)\right] + 3\left[f(7,2) + f(8,1)\right] + 9.$$

$$= f(5,4) + 4(6,3) + 6f(7,2) + 24 + 9$$

$$= f(4,4) + f(5,3) + 4\left[f(5,3) + f(6,2)\right] + 6\left[f(6,2) + f(7,1)\right] + 33$$

$$= f(4,4) + 5f(5,3) + 10f(6,2) + 42 + 33.$$

$$= f(3,4) + f(4,3) + 5\left[f(4,3) + f(5,2)\right] + 10\left[f(5,2) + f(6,1)\right] + 75$$

$$= f(3,4) + 6f(4,3) + 15f(5,2) + 60 + 75.$$

$$= f(2,4) + f(3,3) + 6\left[f(2,3) + f(4,2)\right] + 15\left[f(4,2) + f(5,1)\right] + 135$$

$$= f(2,4) + 7f(3,3) + 21f(4,2) + 210.$$

$$= f(1,4) + f(2,3) + 7\left[f(2,3) + f(3,2)\right] + 21\left[f(3,2) + f(4,1)\right] + 210.$$

$= 1 + 8f(2,3) + 28f(3,2) + 294.$

$= 1+8\left[f(1,3) + f(2,2)\right] + 28\left[f(2,2) + f(3,1)\right] + 294$

$= 1+8+36f(2,2) + 378$

$= 1+8+36 \times 3 + 378$

$= 495.$

2.

Soln :

Given, $x+y+z+w = 8$

Here, $n=4, \quad k=8$

$$f(n,k) = \frac{(n+k-1)!}{(n-1)!\,k!}$$

$$f(4,8) = \frac{(4+8-1)!}{(4-1)!\,8!} = \frac{11!}{3!\,8!}$$

$$= 165.$$

3. let $h(n,k)$ denote the no. of ways which k indistinguishable balls can be coloured with n colours so that there is denote one ball of each colour. Prove that,

i) $h(n,k) = 0$ if $n > k$.

Soln:

$h(n,k) = 0$ if $n > k$.

$h(n,k)$ is the no. of colouring of the k balls with n colours. So that there atleast one ball of each colour.

If the no. of colour is greater the no. of balls, then some colour is not used to one of the balls.

$\therefore h(n,k) = 0$ if $n > k$.

ii) $h(n,k)$ is the coefficient of x^k in $(x, x^2+x^3+\dots)^n$

Soln: consider the expression $(x+x^2+\dots)(x+x^2+\dots)$ $(x+x^2+\dots)$ where there are n brackets of different powers of x are obtained. Each power of x occuring the no. of times. The above expression that x^k is obtained by choosing the term x^{t_1} in the first bracket and x^{t_2} in the second bracket and so on which the condition that,

$$t_1+t_2+\dots\dots+t_k+t_n = k.$$

$\therefore h(n,k) =$ The coefficient of x^k in $(1+x^2+\dots)^n$

$\therefore h(n,k)$ is the coefficient of x^k in $(x+x^2+\dots)^n$

iii) $h(n,k)$ is the coefficient of x^{k-n} in $(1-x)^{-n}$.

Soln: $h(n,k)$ is the coefficient of x^k in $(x+x^2+\dots)^n$

$h(n,k)$ is the coefficient of x^k in $(1+x+\dots)^n$

$h(n,k)$ is the coefficient of x^k in $x^n(1-x)^n$.

$h(n,k)$ is the coefficient of x^{k-n} in $(1-x)^{-n}$

iv) $h(n,k) = f(n,k-n)$.

Soln: $h(n,k) = f(n,k-n)$

$h(n,k)$ is the coefficient of x^{k-n} in $(1-x)^{-n}$ ——— ①

$f(n,k)$ is the coefficient x^k in $(1-x)^{-n}$

$f(n,k-n)$ is the coefficient x^{k-n} in $(1-x)^{-n}$ ——— ②

From ① and ② are equal

$\therefore h(n,k) = f(n,k-n)$.

v) $h(n,k) = \dfrac{(k-1)!}{(n-1)!\,(k-n)!}$ if $n \leq k$. hence find $f(5,10)$.

Soln:

$$h(n,k) = f(n, k-n)$$

$$= \frac{(n+k-n-1)}{(k-n)!\,(n-1)!}$$

$$= \frac{(k-1)!}{(k-n)!\,(n-1)!}$$

$$h(5,10) = \frac{9!}{5!\,4!}$$

$$= 126.$$

Permutations:

Permutation is an ordered arrangements in the given objects.

Let $P(n)$ denote the no. of permutation of n objects.

(For example)

consider $n = 3$.

we have 3 objects A, B, C.

$P(3)$ is the no. of permutation of 3 objects.

$P(3) =$ ABC, ACB, CBA, CAB, BCA, BAC.

Theorem: 1

Prove that $P(n) = n!$ (or) The no. of Permutation of n objects in $n!$

Soln:

Given n objects we have to arrange them in n places.

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The first place can be filled with any one of the n objects in n ways and the remaining $(n-1)$ places can be filled with $(n-1)$ objects in $(n-1)$ ways.

$$P(n) = n\,P(n-1) \quad \text{——— (1)}$$

Now,

$P(1)$ = The no. of permutation of 1 objects.

Thus,
$$\begin{aligned} P(n) &= nP(n-1) \\ &= n(n-1)P(n-2) \\ &= n(n-1)(n-2)P(n-3) \\ &= n(n-1)(n-2)\ldots 2P(1) \\ &= n(n-1)(n-2)\ldots 2.1. \end{aligned}$$

Example : 1

A breakfast competition lists to properties of a new make of car and ask the order to plane there properties in order of importance.

i) How many orderings are possible?

ii) How many would be possible of the first and 10^{th} places where already specified.

Soln:

i) 10! ordering is possible.

ii) 8 properties are left to be ordered.

This can be done in 8! ways.

How many 9 digit number can be obtained by using each of the digits 1, 2, 3 .. 9 exactly once? How many of these are bigger than 500,000,000.

Soln

(i) The no. of ways of arranging to form a 9 digit number using a digit exactly once in 9!

(ii) For the number to be greater than 500,000,000 the first digit must be 5 or 6 or 7 or 8 or 9.

Since the first digit is fixed. The remaining 8 digit can be filled in 8! ways. the requir number of ways $= 5 \times 8!$

$5 \times 8!$ numbers are bigger than 500,000,000

2) How many permutations are there of the 26 letters of the alphabets in which the 5 vowels are in consecutive places.

consider the 5 vowels as a single set.
Then we have 22 letter this can be arranged in 22!

Corresponding to each vowels there are 15 different ordering.

The required no. of ways $= 5!\ 22!$

4) The name of the 12 months of the year are listed in the random order. Given that may and June are not next to each other. How many possible list are there.

soln.

Total no. of permutation of all 12 months is 12!

consider may and June are one unit then there are 11 months.

The total no. of permutation is 11!

Now, may & June can be arrange next to each other 2! ways.

$\therefore$ The no. of permutation of may & June in each other 2! 11!

The required permutation $= 12! - (2!\,11!)$

$= 11!\,(12-2)!$

$= 11!\,10!$

ordered selection.

Let $P(n,r)$ denote the no. of ways of listing r objects choosen from n objects.

The first object of the list are be choosen in n ways and then $(n-1)$ of the remaining objects choosen. The no. of ways of choosing $(r-1)$ objects from $(n-1)$ objects.

$\therefore P(n,r) = nP(n-1, r-1)$

$= n(n-1)P(n-2, r-2).$

$= n(n-1)(n-2)\,P(n-3, r-3)..$

$= n(n-1)(n-2)(n-3)\ldots n-(r-2),$

$P(n-(r-1), r-(r-1)).$

Multiply and divide by $1.2 \ldots (n-r).$

$$P(n,r) = \frac{n!}{(n-r)}$$

12 people to choose from. How many different seating are possible?

Soln.

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(12,5) = \frac{12!}{(12-5)!} = \frac{12!}{7!}$$

$$= 95040.$$

P.2 30 girls, including Miss U.K enter a Miss world competition the frist 6places are annonced.

i) How many different annoncement are possible.

(ii) How many. if Miss U.k is discussed of a place in the first 6.

Soln.

i) we have to choose 6 girls out of 30 girls. This can be done in P(30,6) ways

$P(30,6) = \frac{30!}{24!}$

(ii) The no of placing which do not included miss u.k is (29, 6) ways.

The required place is $P(30,6) - P(29,6)$

$= \frac{30!}{24!} - \frac{29}{23!} = \frac{29!}{23!}\left(\frac{30}{24} - 1\right).$

$= \frac{29!}{23!}\left(\frac{30-24}{24}\right) = \frac{29!}{23!}(6/24)$

$= \frac{29!}{23!}(1/4)$

---

1) Prove that $f(n,k) = C(n+k-1, n-1)$.

Soln.

$$f(n,k) = \frac{(n+k-1)!}{(n-1)!\,(n+k-1-n+1)!}$$

$$= \frac{(n+k-1)!}{(n-1)!\,k!} \quad -①.$$

$$\binom{n+k-1}{n-1} = \frac{(n+k-1)!}{(n-1)!\,k!} \quad -②.$$

from ① and ②.

$f(n,k) = C(n+k-1,\ n-1)$.

Theorem: 1.

$nCr = nC(n-r)$ where $0 \le r \le n$.

proof

$$nCr = \frac{n!}{r!\,(n-r)!} \quad -①.$$

$$nC(n-r) = \frac{n!}{(n-r)!\,(n-n+r)!}$$

$$= \frac{n!}{(n-r)!\,r!} \quad -②.$$

from ① and ②.

$nCr = nC(n-r)$

Theorem: 2

Prove that $nC_{n-2} = nC_2 = \frac{1}{2}\, n(n-1)$.

$$nC_{n-2} = n! / (n-n+2)!\ (n-2)!$$

$$= \frac{(n-2)!\ (n-1)n}{2!\ (n-2)!}$$

$$nC_2 = \frac{n!}{(n-2)!\ 2!} = \frac{(n-2)!\ (n-1)n}{(n-2)!\ 2!} = \frac{1}{2}!\,(n-1)n$$

$$= \frac{1}{2}\, n(n-1) \quad -②.$$

From ① and ②.

$nC_{n-2} = nC_2 = \frac{1}{2}\, n(n-1)$.

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Theorem 3. Prove that $nC_{n-2} = nC_2 = \frac{1}{2} n(n-1)$.

$$nC_{n-2} = \frac{n!}{(n-2)!\,(n-n+2)!}$$

$$= \frac{n!}{2!\,(n-2)!} \Rightarrow \frac{(n-2)!\,(n-1)n}{2!\,(n-2)!} = \frac{n(n-1)}{2!} = \frac{1}{2} n(n-1) \rightarrow (1)$$

$$nC_2 = \frac{n!}{(n-2)!\,(2!)} \Rightarrow \frac{(n-2)!\,(n-1)n}{2!\,(n-2)!} = \frac{1}{2} n(n-1) \rightarrow (2).$$

From (1) & (2), we get,

$$nC_{n-2} = nC_2 = \frac{1}{2} n(n-1).$$

Theorem 1.4. Let n be a positive integer then $(1+x)^n$ is expanded as the sum of the powers of x, the coefficient of x^r is nC_r (or) $\binom{n}{r}$.

Consider $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)(n-2)}{n!} \cdot x^n$.

$$= \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n.$$

A term x^r is obtained by choosing r of the brackets selecting the term x from each of the r brackets & 1 from the remaining $(n-r)$ brackets.

Thus the no. of times x^r is obtained is just the no. of ways are choosing r of the n brackets. ie., nC_r.

$\therefore$ The coefficient of x^r in $(1+x)^n$ in nC_r.

Pascal's Triangle:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \frac{n(n-1)(n-2)(n-3)}{4!}x^4 + \dots$$

Now $(1+x)^0 = 1$.

$(1+x)^1 = 1+x$

$(1+x)^2 = 1 + 2x + \frac{2(1)}{2!}x^2 \Rightarrow 1 + 2x + x^2$.

$(1+x)^3 = 1 + 3x + 3x^2 + \frac{3(2)(1)}{3!}x^3 \Rightarrow 1 + 3x + 3x^2 + x^3$.

$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$.

⋮

The coefficient in the expansion are,

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

This array is known as pascal's triangle.

The $(n+1)^{th}$ row gives the number $nC_0, nC_1, nC_2, \dots, nC_n$.

Note: i) Each row read the same forwards & backwards also.

ii) Each number in the array is the sum of the 2 numbers internatively above it.

Theorem 5. Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$

$$R.H.S = \binom{n-1}{r-1} + \binom{n-1}{r}$$

$$= \frac{(n-1)!}{(n+r+1)!\,(r-1)!} + \frac{(n-1)!}{(n-r-1)!\,r!}$$

$$= \frac{(n-1)!}{(n-r)!\,(r-1)!} + \frac{(n-1)!}{(n-r-1)!\,r!}$$

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left[\frac{1}{(n-r)} + \frac{1}{r}\right] = \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left[\frac{r+n-r}{r(n-r)}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left[\frac{n}{r(n-r)}\right]$$

$$= \frac{n!}{(n-r)!\,r!}$$

$$\binom{n-1}{r-1} + \binom{n-1}{r} = \binom{n}{r}$$

Note: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$

$$= \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n.$$

Theorem 6: Binomial Theorem

If n is any positive integer. Then $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n}b^n$ (or)

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r.$$

Proof:

$$(a+b)^n = a^n(1+b/a)^n$$

$$= a^n\sum_{r=0}^{n}\binom{n}{r}\left(\frac{b}{a}\right)^r = a^n\sum_{r=0}^{n}\binom{n}{r}\left(\frac{b^r}{a^r}\right)$$

$$= \sum_{r=0}^{n}\binom{n}{r}b^r a^{n-r}.$$

$$\therefore (a+b)^n = \sum_{r=0}^{n}\binom{n}{r}b^r a^{n-r}.$$

Theorem 7. If n is positive Integer Then $(1-x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$ (or)

$$(1-x)^n = \sum_{r=0}^{n}\binom{n}{r}(-1)^r x^r.$$

The result followed from the Binomial theorem by choosing $a=1$ & $b=(-x)$.

$$(1-x)^n = \sum_{r=0}^{n}\binom{n}{r}(-x)^r(1)^{n-r}.$$

$$(x+y)^r = \sum_{r=0}^{n}\binom{n}{r}(-1)^r(x)^r(1)^{n-r}$$

$$\therefore (1-x)^n = \sum_{r=0}^{n}\binom{n}{r}(-1)^r x^r.$$

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Result:

$$e^{x} \cdot e^{y} = e^{x+y} \quad (\text{or}) \quad \exp(x) \cdot \exp(y) = \exp(x+y)$$

Proof:

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^{y} = 1 + \frac{y}{1!} + \frac{y^2}{2!} + \cdots = \sum_{k=0}^{\infty} \frac{y^k}{k!}$$

L.H.S

$$e^{x} \cdot e^{y} = \sum_{k=0}^{\infty} \frac{x^k}{k!} \cdot \sum_{k=0}^{\infty} \frac{y^k}{k!} = \sum_{k=0}^{\infty} \frac{x^k}{k!} \cdot \frac{y^k}{k!}$$

$$= \sum_{k=0}^{\infty} a_k \cdot b_k \text{ where } a_k = \frac{x^k}{k!} \text{ and } b_k = \frac{y^k}{k!}$$

$$e^{x} \cdot e^{y} = \sum_{k=0}^{\infty} c_k \text{ where } c_k = a_k b_0 + a_{k-1} b + a_{k-2} b_2 + \cdots + a_0 b_k$$

$$e^{x} \cdot e^{y} = \sum_{k=0}^{\infty} a_k \text{ where}$$

$$c_k = a_k b_0 + a_{k-1} b_1 + \cdots + a_0 b_k$$

$$= \frac{x^k}{k!} \frac{y^0}{0!} + \frac{x^{k-1}}{(k-1)!} \frac{y!}{1!} + \frac{x^{k-2}}{(k-2)!} \cdot \frac{y^2 x}{2!} + \cdots + \frac{x^2}{0!} \cdot \frac{y^k}{k!}$$

multiply and divide by k:

$$= \frac{1}{k!} \left[\frac{k!\, 2^k}{k!} + k! \frac{x^{k-1}}{(k-1)!} y + \cdots + k! \frac{y^k}{k!} \right]$$

$$= \binom{k}{0} x^k + \binom{k}{1} x^{k-1} y + \cdots + \binom{k}{k} y^k$$

$$c_k = \frac{1}{k} (x+y)^k \Rightarrow e^{x} \cdot e^{y} - \frac{1}{k!} \sum_{k=0}^{\infty} (x+y)^k.$$

2] $(x+y)^7$

Soln:

$$(x+y)^7 = \sum_{r=0}^{7} \binom{7}{r} y^r x^{n-r}$$

$$= \binom{7}{0} x^7 y^0 + \binom{7}{1} x^6 y^1 + \binom{7}{2} x^5 y^2 + \binom{7}{3} x^4 y^3$$

$$+ \binom{7}{4} x^3 y^4 + \binom{7}{5} x^2 y^5 + \binom{7}{6} x y^6 + \binom{7}{7} y^7$$

$$= x^7 + 7x^6 y + \frac{7\times 6}{1\times 2} x^5 y^2 + \frac{7\times 6\times 5}{1\times 2\times 3} x^4 y^3 + \frac{7\times 6\times 5}{1\times 2\times 3} x^3 y^4$$

$$+ \frac{7\times 6}{1\times 2} x^2 y^5 + 7xy^6 + y^7$$

$$(x+y)^7 = x^7 + 7x^6 y + 21x^5 y^2 + 35x^4 y^3 + 35x^3 y^4 + 21x^2 y^5 + 7xy^6 + y^7$$

3] Prove that $\binom{n+1}{3} - \binom{n-1}{3} = (n-1)^2$

Soln:

$$\binom{n+1}{3} - \binom{n-1}{3} = \frac{(n+1)!}{3!(n-2)!} - \frac{(n-1)!}{3!(n-4)!}$$

$$= \frac{(n+1)!}{3!(n-2)!} - \frac{(n-1)!}{3!(n-4)!} = \frac{(n-1)!}{3!(n-4)!}\left[\frac{n(n+1)}{(n-3)(n-2)}\right]$$

$$= \frac{(n-1)!}{3!(n-4)!}\left[\frac{(n(n+1)) - (n-3)(n-2)}{(n-3)(n-2)}\right] = \frac{(n-1)!}{3!(n-4)!}\left[\frac{n^2 - n - n^2 + 5n - 6}{(n-3)(n-2)}\right]$$

$$= \frac{(n-1)!}{3!(n-4)!}\left[\frac{6n-6}{(n-3)(n-2)}\right] = \frac{(n-1)!}{6(n-4)!}\left[\frac{6(n-1)}{(n-3)(n-2)}\right]$$

$$= \frac{(n-4)!(n-3)(n-2)(n-1)}{(n-4)!}\left[\frac{(n-1)}{(n-3)(n-2)}\right]$$

$$= (n-1)^2$$

4] Theorem :- If n is any positive integer, then $(1-x)^{-n} = \binom{n}{0} + \binom{n}{1} x + \binom{n+1}{2} + \cdots + \binom{n+r-1}{r} x^r$ (or) $(1-x)^{-n} = \sum_{r=0}^{\infty} \binom{n+r-1}{r} x^r$

Proof:

Let us prove that the coefficient of x^r is $\binom{n+r-1}{-r}$

$F(n,r)$ = The coefficient of x^r in $(1-x)^{-n} \rightarrow ①$

Bell approach.

$F(x,r) = \binom{n+r-1}{n-1} \rightarrow ②$

From ① & ②

The coefficient of r in $(1-x)^{-n}$ is $\binom{n+r-1}{r}$

$$(1-x)^{-n} = \binom{n+r-1}{n-r} = \frac{(n+r-1)!}{(n-1)!\,(n+r-1-n+1)!}$$

$$= \frac{(n+r-1)!}{(n-1)!\,(r)!} = \binom{n+r-1}{r}$$

5] An eight man committee is to be formed from a group of ten welshman and 15 English man. In how many ways can be committee we chosen.

(i) The committee must contain 4 of each nationality

(ii) There must be atleast two welsh mans.

(iii) There must be more welshman than English man.

Soln:

(i) Since the committee must contain 4 of each nationality.

The no. of ways forming the committee $= \binom{10}{4}\binom{15}{4}$

$= 210 \times 1365$

$= 286650$

(ii) Since the committee must contain more welshmen than English man.

$\therefore$ The no. of ways of forming the committee.

$$= \binom{10}{8}\binom{15}{0} + \binom{10}{7}\binom{15}{1} + \binom{10}{6}\binom{15}{2} + \binom{10}{5}\binom{15}{3}$$

$= 138555.$

(iii) Since the committee must contain atleast 2 welshman.

$\therefore$ The required no. of ways of forming the committee =

Total no. of ways - no. of forming the committee almost two welshmen.

$$=\binom{2n}{8}-\left\{\binom{10}{0}\binom{15}{8}+\binom{10}{1}\binom{15}{7}\right\}$$

$$=1081575-(6435+64350)$$

$$=1010790.$$

b) By using the identity $(1+x)^{2n}=(1+x)^n(1+x)^n$ and considering the coefficient of x^n on both sides. Prove that $\binom{2n}{n}=\binom{n}{0}^2+\binom{n}{1}^2+\dots+\binom{n}{n}^2$ and verify this in the case $n=5$.

Sol:

$$(1+x)^{2n}=\binom{2n}{0}+\binom{2n}{1}x+\binom{2n}{2}x^2+\dots+\binom{2n}{n}x^n+\dots+\binom{2n}{2n}x^{2n}$$

Here the coefficient of x^n is $\binom{2n}{n}\rightarrow①$

$$(1+x)^n(1+x)^n=\left(\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\dots+\binom{n}{n}x^n\right)\left(\binom{n}{0}+\binom{n}{1}x+\dots\binom{n}{n}x^n\right)$$

Here the coefficient of x^n is

$\binom{n}{0}\binom{n}{n}+\binom{n}{1}\binom{n}{n-1}+\binom{n}{2}\binom{n}{n-2}+\dots+\binom{n}{0}$ By theorem 1,

$$\binom{n}{r}=\binom{n}{n-r}$$

$$\binom{n}{n}=\binom{n}{0}$$

$$\binom{n}{n-1}=\binom{n}{1}$$

The coefficient of x^n is.

$$\left[\binom{n}{0}\binom{n}{n}+\binom{n}{1}\binom{n}{n-1}+\binom{n}{2}\binom{n}{n-2}+\dots+\binom{n}{0}\binom{n}{0}\right]\rightarrow①$$

$$=\binom{n}{0}^2+\binom{n}{1}^2+\binom{n}{2}^2+\dots+\binom{n}{n}^2\rightarrow②$$

From ① & ② we get,

$$\binom{2n}{n}=\binom{n}{0}^2+\binom{n}{1}^2+\binom{n}{2}^2+\dots+\binom{n}{n}^2$$

Put $n=5$

L.H.S. $\binom{2n}{n}=\binom{10}{5}=252\rightarrow①$

R.H.S $\Rightarrow\binom{n}{0}^2+\binom{n}{1}^2+\binom{n}{2}^2+\dots+\binom{n}{n}^2=\binom{5}{0}^2+\binom{5}{1}^2+\dots\binom{5}{5}^2$

$$=1+25+100+100+25+1=252\rightarrow②$$

From ① & ②

$$\binom{2n}{n}=\binom{n}{0}^2+\binom{n}{1}^2+\dots+\binom{n}{n}^2$$

problem 5:

A team of 11 players is to be chosen from a pool of 15 i) How many team selections are possible? ii) How many if one of the 15 has already been applied captain and must play.

Soln: i) We have to choose out of 15 players This can been done in $15C_{11}$ ways.

$$\therefore 15C_{11} = 1365$$

ii) suppose one of the 15 has already been appointed as captain and must play.

Then the required ways $14C_{10}$ ways

$$\therefore 14C_{10} = 1001$$

problem: 3

For each day of the 5 day working week can choosen any one of 4 newspaper to read in the train. How many different buys are possible in a week?

Soln: one can buy the same newspaper. 1 to 5 days. The no of ways to choose the newspaper in the day = 4.

$\therefore$ The required no of ways $= 4 \times 4 \times 4 \times 4 \times 4$

$= 4^5$ ways.

Ex: 1 Evaluate $P(7,4)$, $P(8,2)$, $P(9,5)$

Soln: $P(7,4) = 7!/3! = 840.$

$p(8,2) = 8!/6! = 56$

$p(9,5) = 9!/4! = 15120$

2 A car registeration number is to consist of 3 letters followed by a number between 1 and 199 how many car numbers are possible?

soln:

The no of ways to choose each of the letters $26 \times 26 \times 26$

The no of ways to choose the number between 1 & 199 = 999.

$\therefore$ The required no of ways = $26 \times 26 \times 26 \times 999$

problem: 3

Tom has 75 books but enough room on his room selves for only 20 in how many ways can be filled his shelf?

soln:

The no of ways to fill his self = The no of ways to choose 20 books

The required no of ways = $p(75,20)$

$= 75!/55!$

problem: 4.

How many numbers between 100 & 3000 can be formed from the digit 1, 2, 3, 4, 5 If repetition of digit is i) allowed. ii) not allowed.

Soln: i) Allowed.

The no. of should be between 1000 and 3000

The first digit 1 or 2. This can be done in 2 ways

Since repetition is allowed, second digit can be choosen in 5 ways.

III-ly 3rd & 4th digit can be chosen in 5 ways.

$\therefore$ The required no. of ways $= 2 \times 5 \times 5 \times 5$

$= 250$

ii) Not allowed.

The no. of should be between 1000 & 3000

The first digit 1 or 2. This can be done in 2 ways.

Since repetition is not allowed, The 2nd digit can be choosen in 4 ways. and 3rd digit can be choosen in 3 ways & 4th digit can be choosen in 2 ways.

$\therefore$ The required no of ways $= 2 \times 4 \times 3 \times 2$.

$= 48$.

In how many ways can 4 types we put on a car?

Soln: The no. of ways of placing 4 types in a car $= 4!$

problem 7:

prove that $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n} = 2$

Proof: $(1+x)^n = \binom{n}{0}+\binom{n}{1}x+\cdots+\binom{n}{n}x^n$

put $x=1$

$$2^n = \binom{n}{0}+\binom{n}{1}+\cdots\binom{n}{n}$$

problem 8:

prove that $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\cdots+(-1)^n\binom{n}{n}=0$

Soln:

$$(1-x)^n = \binom{n}{0}-\binom{n}{1}x+\binom{n}{2}x^2+\cdots+(-1)^n\binom{n}{n}$$

put $x=1$.

$$0 = \binom{n}{0}-\binom{n}{1}+\binom{n}{2}+\cdots+(-1)^n\binom{n}{n}$$

problem: 9.

prove that $\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\cdots+n\binom{n}{n}=0.$

Soln:

$$(1+x)^n = \binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\cdots+\binom{n}{n}x^n.$$

$$n(1+x)^{n-1} = \binom{n}{1}+2\binom{n}{2}x+\cdots+n\binom{n}{n}x^n.$$

$$n\,2^{n-1} = \binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\cdots+n\binom{n}{n}.$$

Problem: 10

Show that $nP_r + r(nP_{r-1}) = (n+1)P_r$.

Proof:

L.H.S

$$nP_r + r(nP_{r-1}) = \frac{n!}{(n-r)!} + r\left[\frac{n!}{(n-r+1)!}\right]$$

$$= \frac{n!}{(n-r)!}\left[1 + \frac{r}{n-r+1}\right]$$

$$= \frac{n!}{(n-r)!}\left[\frac{n-r+1+r}{n-r+1}\right]$$

$$= \frac{n!}{(n-r)!}\left[\frac{n+1}{n-r+1}\right]$$

$$= \frac{(n+1)!}{(n-r+1)!} \quad \text{——} (1)$$

$$nP_r + r(nP_{r-1}) = (n+1)P_r$$

Find the no. of ways of choosing 9 different letter from the word.

i) MATHEMATICS   ii) COIMBATORE

Soln:

In MATHEMATICS there are different letter

The no. of way of choosing 2 different

The no. of ways different letter from the coimbatore is $9c_2 = 36$

Pbm 1:

Prove that $\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \dots \binom{n+n}{n} = \binom{n+m+1}{m}$

Proof:

It is clearly

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

R.H.S.

$$\binom{n+m+1}{m} = \binom{n+m}{m-1} + \binom{n+m}{m}$$

$$= \binom{n+m-1}{m-2} + \binom{n+m-1}{m-1} + \binom{n+m}{m}$$

$$= \binom{n+m-2}{m-3} + \binom{n+m-2}{m-2} + \binom{n+m-1}{m-1} + \binom{n+m}{m}$$

$$= \binom{n+m-m}{m-m} + \binom{n-m-(m-1)}{m-(m-1)}$$

$$= \binom{n}{0} + \binom{n+1}{1} + \dots + \binom{n+m}{m}$$

Pbm: 13

Show that in each row of Pascal's triangle the largest no is the middle one ie) show that $\binom{n}{r} > \binom{n}{r-1}$ if $r < \frac{1}{2}(n+1)$

Soln.

To Prove

$$\binom{n}{r} > \binom{n}{r-1}$$

$$\binom{n}{r} = \frac{n!}{(n-r)\,r!} \quad \text{—①}$$

$$\binom{n}{r-1} = \frac{n!}{(n-r+1)!\,(r-1)!}$$

multiply and divide by r

$$\binom{n}{r-1} = \frac{n!\,r}{r!\,(n-r+1)\,(n-r)!}$$

$$\binom{n}{r-1} = \frac{n!}{(n-r)!\,r!}\left(\frac{r}{n-r+1}\right) \text{—②}$$

given.

$$r < \tfrac{1}{2}(n+1)$$

$$\Rightarrow 2r < n+1$$

$$\Rightarrow r - r < n - r + 1$$

$$\Rightarrow r < n - r + 1$$

$$\Rightarrow \frac{r}{n-r+1} < 1$$

apply eqn ② we get $\binom{n}{r} > \binom{n}{r-1}$

Pbm: 14

$$\binom{n}{r}^2 > \binom{n}{r-1}\binom{n}{r+1}$$

Soln:

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$$\binom{n}{r}^2 = \left(\frac{n!}{(n-r)!\,r!}\right)^2$$

$$= \left(\frac{(n!)^2}{(r!)^2\,(n-r)!^2}\right) \text{—①}$$

$$\binom{n}{r-1}\binom{n}{r+1} = \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{(n!)^2}{(n-r+1)(n-r)!\,(r-1)!} \times \frac{1}{(n-r-1)!\,(r+1)!\,(r-1)!}$$

Multiply and divide by r and $(n-r)$

$$\binom{n}{r-1}\binom{n}{r+1} = \frac{(n!)}{(n-r+1)(n-r)!\,(r-1)!} \times \frac{n(n-r)}{(n+r-1)!\,(n-r)}$$

$$= \frac{n!\,r}{(n-r)!\,(n-r+1)!\,(r-1)!} \times \frac{n!\,(n-r)}{(n-r)!\,r!\,(r+1)!}$$

given,

$$nr - r^2 < nr - r^2 + (n+1)$$

$$\frac{nr - r^2}{nr - r^2 + (n+1)} < 1$$

Apply in eqn ① we get

$$\binom{n}{r}^2 > \binom{n}{r-1}\binom{n}{r+1}$$

Pbm: 7

In how many ways can a 5 letter word be formed from an alphabet of 26 letter if repetition are allowed ii) not allowed

Soln:-

i) Allowed

The first letter can be chosen in 26 ways

Since a polition a - the second

III ly, 3rd, 4th, 5th letter can choosen. 26

The required no. of ways.

$= 26 \times 26 \times 26 \times 26 \times 26$

$= 11881376$

ii) not allowed.

The Fist letter can be chosen is 26 ways.

Since repetition is not allowed the Second letter can be choosen in 25 ways.

and 3rd letter can be choosen in 24 ways.

and 4th letter can be choosen in 23 ways

and 5th letter can be choosen in 22 ways

The required no. of ways $= 26 \times 25 \times 24 \times 23 \times 22$

$= 7893600$

Pbm: 6

There are k stations on a Particular railway If it is possible the travel From any attation to any other how many difference Fourneys are possible?

Soln.

Given there are k Station and it is possible to travel From any station to anyother.

k ways to Select the The required number is $k(k-1)$

Prbm: 8

The binary sequence of length n is a string of n digits each of which is 0 (or) How many sequence are there? List all those of length 4

Soln:

Given a binary sequence of length n is a string of digits each of which is 0 (or) 1

The first digit can be choosen in 2 way and the second digit can be choosen in 2 ways and so on.

The required no. of ways

$= 2 \cdot 2 \ldots 2$ (n times)

$= 2^n$

There are 2^n such sequences.

ii) Suppose the length of the string is 4. Then the sequences is $2^4 = 16$.